

Cross-Layer Scheduling for OFDMA-based Cognitive Radio Systems with Delay and Security Constraints

Xingzheng Zhu, Bo Yang, Cailian Chen, Liang Xue, Xiping Guan, Fan Wu

Abstract—This paper considers the resource allocation problem in an Orthogonal Frequency Division Multiple Access (OFDMA) based cognitive radio (CR) network, where the CR base station adopts full overlay scheme to transmit both private and open information to multiple users with average delay and power constraints. A stochastic optimization problem is formulated to develop flow control and radio resource allocation in order to maximize the long-term system throughput of open and private information in CR system and ensure the stability of primary system. The corresponding optimal condition for employing full overlay is derived in the context of concurrent transmission of open and private information. An online resource allocation scheme is designed to adapt the transmission of open and private information based on monitoring the status of primary system as well as the channel and queue states in the CR network. The scheme is proven to be asymptotically optimal in solving the stochastic optimization problem without knowing any statistical information. Simulations are provided to verify the analytical results and efficiency of the scheme.

Index Terms—Cognitive radio, physical-layer security, delay-aware network, full overlay, cross-layer scheduling.

I. INTRODUCTION

THE emergency of high-speed wireless applications and increasing scarcity of available spectrum remind researchers of spectrum utilizing efficiency. The concept of CR provides the potential technology in increasing spectrum utilizing efficiency [1], [2] because CR allows unlicensed users (also known as secondary users (SUs)) to access some spectrum which is already allocated to primary user (PU) or licensed user who has the authority to access the spectrum by spectrum sensing [3], [4]. As another promising technology of high speed wireless communication system, OFDMA is a candidate for CR systems [1] due to its flexibility in allocating spectrum among SUs [5]. Hence, OFDMA-based CR networks are catching great attention [6], [7]. This paper focuses on an OFDMA-based CR network without loss of generality.

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In order to exploit the capacity of the whole OFDMA-based CR system, this paper aims at maximizing the secondary network capacity in consideration of the whole system transmission efficiency. Thus, the following three main issues should be considered.

Firstly, an efficient spectrum sharing scheme is essential for exploiting the unused spectrum in OFDMA-based CR network. When a SU wants to access some spectrum, it must ensure that the spectrum is not accessed by any PU or adapt its parameter to limit the interference to PU. Both of these two mentioned spectrum utilization manners, known as overlay and underlay schemes, are conservative in some ways, since they ignore the PU's ability to tolerate some inference.

Secondly, due to that CR networks as well as many other kinds of wireless communication systems have a nature of broadcast, security issues at physical layer have always been unavoidable in designing CR systems. Furthermore, to SUs, it is obviously practical that there exist both private and open transmission requirements. Then, the scheduling among these two different kinds of transmission should be considered. In addition, delay performance is an indispensable quality of service (QoS) index in scheduling different transmissions.

Last but not least, the dynamic nature of OFDMA-based CR communication system brings another big challenge. The random arrival of user requests (from both PU and SU) and time-varying channel states renders dynamic resource allocation instead of fixed ones in exploiting the OFDMA secondary network capacity.

Aiming at the above issues, the contributions of this paper are threefold:

- First, this paper adopts a novel full overlay spectrum accessing scheme by exploiting PU's tolerance to interference. Besides, the theoretic proof of full overlay's optimality is given in the presence of both open and private transmissions.
- Second, a joint encoding model is introduced to allow both private and open transmissions towards SUs with the full overlay spectrum sharing scheme. A dynamic resource allocation scheme consisting of flow control and radio resource allocation is developed by solving a formulated stochastic optimization problem under the delay and power constraints.
- Third, the proposed dynamic resource allocation scheme is proven to be close to optimality although its implementation only depending on instantaneous information.

This paper is organized as follows. Section II presents

the related work. In Section III, we introduce the system model and relevant constraints in detail. Section IV formulates the problem. In Section V, we introduce our cross-layer optimization algorithm. We give the performance bound and stability results in Section VI. Two different implementations are proposed in Section VII. In Section VIII, some simulation results are shown. Finally, we conclude this paper in Section IX.

II. RELATED WORK

There have been many works on spectrum sharing in OFDMA-based CR networks [8]–[10]. According to [11], [12], the access technology of the SUs can be divided in two categories: spectrum underlay and spectrum overlay. The first category means that SUs can access licensed spectrum during PUs' transmission, while as is mentioned in [12], this approach imposes severe constraints on the transmission power of SUs such that they can operate below the noise floor of PUs, e.g. in [8], [13], [14]. The second category means that SUs can only access licensed spectrum when the PU is idle, e.g. in [9], [10], [15]–[17]. Considering both these two strategies suffer from some drawbacks, the authors in [18] propose a new cognitive overlay scheme requiring SUs to assess and control their interference impacts on PUs. In general, the cognitive base station (CBS) controls the aggregate interference to primary transmission by allowing SUs to monitor channel quality indicators (CQIs), power-control notifications and ACK/NAK of primary transmission. In this paper, this novel thought is extended into an OFDMA-based CR system.

On the other hand, dynamic resource allocation plays a critical role in exploiting OFDMA network capacity. The overall performance as well as the multiuser diversity of the system can be improved by proper dynamic resource allocation [17], [19]–[25]. Thus, dynamic resource allocation in OFDMA-based CR system has been attracting more attention recently. The corresponding spectrum sharing schemes in [8]–[10] are all realized by dynamic resource allocation.

Besides the interference constraints, the works of delay aware transmission are also quite relative to this paper. Huang and Fang in [26] investigate both reliability and delay constraints in routing design for wireless sensor network. Cui et al. in [27] summarize three approaches to deal with delay-aware resource allocation in wireless networks. A constrained predictive control strategy is proposed in [28] to compensate for network-induced delays with stability guarantee. Those three methods are based on large deviation theory, Markov decision theory and Lyapunov optimization techniques. As to the first two methods, they have to know some statistical information on channel state and random arrival data rate to design algorithm, while these prior knowledge is expensive to get, even unavailable. To overcome this problem, many authors pay attention to Lyapunov optimization techniques. References [29] and [30] investigate scheduling in multi-hop wireless networks and resource allocation in cooperative communications, respectively as two typical applications of Lyapunov optimization in delay-limited system. In this paper, we utilize this tool to dispose the resource allocation problem in OFDMA-based CR networks.

As for secure transmission, Shannon's information theory laid the foundation for information-theoretic security [31] and the concept of wire-tap channel was proposed in [32]. There has been some research on exploiting security capacity in OFDMA network by dynamic resource allocation, such as in [33] and [34]. In CR area, the study of secure transmission from information-theoretic aspect is very limited. Pei et al. in [35] first investigate secrecy capacity of the secure multiple-input single-output (MISO) CR channel. Kwon et al. in [36] utilize the concept of security capacity to explore MISO CR systems where the secondary system secures the primary communication in return for permission to use the spectrum. Both these two works focus on only private message transmission. The security and common capacity of cognitive interference channels is analyzed in [37]. The entire capacity of a MIMO broadcast channel with common and confidential messages is obtained in [38]. The paper [39] considers the problem of optimizing the security and common capacity of an OFDMA downlink system by dynamic resource allocation. This paper further considers the transmissions of private and open flows in CR networks with delay constraints.

III. SYSTEM MODEL

The system model consists of multiple primary links and multiple secondary links as Fig. 1 shows. The total bandwidth B is divided into M subcarriers equally using Orthogonal Frequency Division Multiplexing (OFDM). Assume that $M = B$ holds for simplicity of expression. The subcarrier set of the network is denoted as $\mathbf{M} = \{1, 2, \dots, M\}$ and $m \in \mathbf{M}$ denotes subcarrier index. The downlink case is considered. The primary link is from a single primary base station (PBS) to K PUs. Secondary links are from a common CBS to N SUs. We denote $k \in \{1, 2, \dots, K\}$ and $n \in \{1, 2, \dots, N\}$ as the indexes of PU and SU respectively. The system operates in slotted time, and T is the length of a time slot. Hereafter, $[tT, (t+1)T)$ is just denoted by t for brevity.

The set of subcarriers occupied by PU k on timeslot t is denoted as $\Gamma_k^{PU}(t) = \{\tau_1^k(t), \tau_2^k(t), \dots, \tau_{m^k(t)}^k(t)\}$ where $m^k(t)$ is the number of subcarriers occupied by PU k and $\Gamma_k^{PU}(t) \subseteq \{1, 2, \dots, M\}$. The power set $\mathbf{P}_k^{PU}(t) = \{P_k^m(t) | m \in \mathbf{M}\}$ is the set of transmission power from PBS to PU k , where for $m \in \Gamma_k^{PU}(t)$, $P_k^m(t) > 0$, else $P_k^m(t) = 0$. For brevity, we will omit the time index (t) somewhere in further discussion. $\mathbf{P}^{SU} = \{p_n^m | \forall n, \forall m\}$ denotes the overall SUs power allocation policy set and p_n^m represents the power allocated by CBS to user n in subcarrier m . Denote $\Gamma_n^{SU} = \{\varpi_n^m | \forall m\}$ as the subcarrier assignment policy of SU n , where ϖ_n^m is either 1 representing subcarrier m is assigned to SU n , or 0 otherwise. Then let $\Gamma^{SU} = \{\Gamma_n^{SU}(t), \forall n\}$ be the overall subcarrier assignment policy of secondary network.

Due to the orthogonal properties of OFDMA technology, there exists no mutual influence between every two SUs. However, there exists mutual interference between the primary and secondary networks when PU and SU access in the same subcarrier.

The channel gains include the one of secondary user n on subcarrier m , h_n^m and the one of primary user k

on subcarrier m , H_k^m . The additive white gaussian noise (AWGN) is σ^2 . The corresponding subcarrier gain-to-noise-ratio¹ (C/I) in slot t are thus defined as $a_n^m(t) = \frac{h_n^m(t)^2}{\sigma^2}$ and $A_k^m = \frac{H_k^m(t)^2}{\sigma^2}$ respectively as illustrated in Fig.1. The set $\mathbf{a}(t) = \{A_k^m(t), a_n^m(t), \forall n, \forall m, \forall k\}$ represents the system channel state information (CSI). All channels are assumed to be slow fading, and thus $\mathbf{a}(t)$ remains fixed during one slot and changes between two [40]. In this work, there exists a reasonable assumption that the system CSI is known to BS. As in [41], BS can get full-CSI by utilizing pilot symbols and CSI feedback process. Besides, at the beginning of every slot, PU reports $P_k^m A_k^m$ to PBS. For example, the PU reports a received-signal-strength index to PBS in packets such as RSSI reports. We assume the CBS will listen to the information to derive $P_k^m A_k^m$ before accessing subcarrier m [18], [41].

Denote h_{kS}^m as the cross-link interference channel gain from CBS to PU k on subcarrier m and let $a_{kS}^m = \frac{h_{kS}^m{}^2}{\sigma^2}$. Similarly, denote h_{nP}^m as the cross-link interference channel gain from PBS to SU n on subcarrier m and let $a_{nP}^m = \frac{h_{nP}^m{}^2}{\sigma^2}$. It is assumed that a_{kS}^m and a_{nP}^m can be got by the CBS. a_{kS}^m can be estimated by CBS from the PU feedback signal based on reciprocity. a_{nP}^m can be estimated by SUs through training and sensing and the estimation results are sent to CBS [41]. Beyond that, information about cross-link channel state could also be measured periodically by a band manager either [8], [42].

Compared to pervious work, this paper considers a more complicated and practical situation of SU transmission. The CBS transmits both private and open data to each SU as Fig.2 shows. The private data has security requirement and open data has long-term time-average delay constraint. Instead of that both open and private data have delay constraint, only delay constraints on open transmissions are considered in this paper for simplifying the mathematic expressions, since the handling of delay constraint in secure transmission is totally the same as open transmission. Actually, in real wireless communication systems, there exists some private transmission having no strict delay constraint, e.g. updating contact information in mobile devices. At the beginning of every time slot, random data packets arrive at CBS. CBS decides whether to admit it into the system or not. Besides, CBS is also in charge of resource allocation to assign power and subcarriers among SUs. CBS utilizes the information of data queue and CSI to allocate resources. The system performance can be optimized and the queuing delay of open data can be ensured to fulfill by flow control and resource allocation.

In the side of CBS, the amount of open data packet of SU n , $D_n^o(t)$, and private data, $D_n^p(t)$ that arrive at CBS during slot t are independent identically distributed (i.i.d) stochastic processes, e.g. Bernoulli processes, with the long-term average arrival rates λ_n^o and λ_n^p , and their upper bounds are μ_{max} and D_{max} , respectively. These packets can not be transmitted to target users instantaneously due to the time-varying channel conditions and they are enqueued at the CBS. However, only parts of these packets are admitted into each queue towards

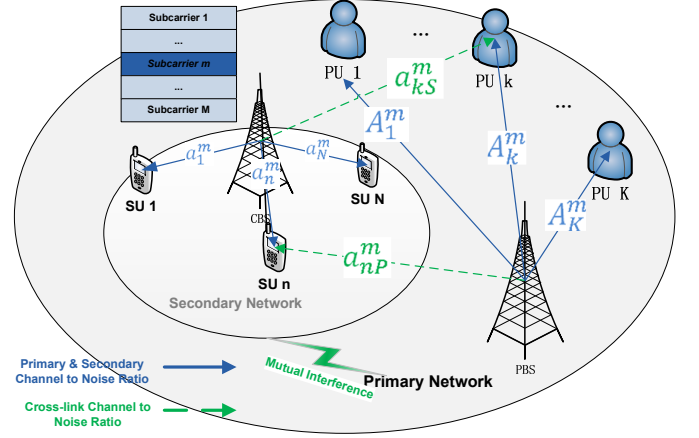


Fig. 1. General network model

each user for stability reason to be specified later. The amounts of open and private data admitted by respect queues are $T_n^o(t)$ and $T_n^p(t)$ and CBS is in charge of determining $T_n^o(t)$ and $T_n^p(t)$ according to a certain principle which would be specified in Section V.

A. Capacity model

In OFDMA-based CR networks, SU and PU can access in the same subcarrier with mutual interference. However, due to the characteristic of OFDMA networks, each subcarrier can not be assigned to more than solitary user in any secondary or primary network. Thus the following formulation is set to ensure the limitation in CBS:

$$0 \leq \sum_{n=1}^N \varpi_n^m \leq 1, \quad \forall m \quad (1)$$

CBS will realize the occupied subcarrier set $\Gamma_k^{PU} = \{m | P_k^m > 0, \forall m\}$, and we denote $\Gamma_{SU} = \{1, 2, \dots, M\} - \bigcup_{k=1}^K \Gamma_k^{PU}$. Thus the transmission rates of PU and SUs can be analysed by dividing M subcarriers into two parts: one is $m \in \bigcup_{k=1}^K \Gamma_k^{PU}$ where there exists interference between PU and SUs; another is $m \in \Gamma_{SU}$ which means SUs can access these subcarriers without influencing primary link. Thus according to information theory the transmission rate of PU k on subcarrier m is:

$$R_k^m = \begin{cases} \log_2(1 + \frac{P_k^m A_k^m}{1 + a_{kS}^m P_{n'}^m}) & m \in \Gamma_k^{PU}, n' \in \tilde{\Gamma}_m \\ 0 & m \in \Gamma_{SU} \end{cases}$$

where $\tilde{\Gamma}_m$ is the set of SUs accessing subcarrier m . Furthermore, since in secondary network, only one SU can access one subcarrier, n' is the only one element in set $\tilde{\Gamma}_m$.

It should be noticed that the total transmission rate in an OFDMA network equals to the sum rates on all subcarriers. So the transmission rate of PU is:

¹Also called gain-to-noise-plus-interference-ratio when SU and PU access in the same subcarrier.

$$R_k^{PU} = \sum_{m \in \Gamma_k^{PU}} R_k^m \quad (2)$$

The channel capacities of SU n on subcarrier m can be expressed as:

$$C_n^m = \begin{cases} \log_2(1 + \frac{p_n^m a_n^m}{1 + P_{k'}^m a_{n,P}^m}) & m \in \bigcup_{k=1}^K \Gamma_k^{PU}, k' \in \hat{\Gamma}_m \\ \log_2(1 + p_n^m a_n^m) & m \in \Gamma_{SU} \end{cases}$$

where $\hat{\Gamma}_m$ is the set of PUs accessing subcarrier m . Furthermore, since only one PU can access one subcarrier, k' is the only one element in set $\hat{\Gamma}_m$. Denote $R_n^{SU} = \sum_m C_n^m$ as the sum transmission rate of SU n without consideration of security.

By introducing the joint transmission model, open and private data of one SU can be transmitted simultaneously. Open message is jointly encoded with security message as random codes. In this way, although open message may be decoded by eavesdroppers, security message would be perfectly secure if the channel fading is properly utilized [43]. According to the theory of physical-layer security [34], if the transmission rate of private data is less than *security capacity*, the proposed joint-encoding model can at least realize physical-layer security in theory. [44], [45] propose physical-layer security realization applications using error correcting codes and pre-processor, which lays the foundation of realizing physical-layer security of the joint encoding model. For each SU, CBS makes decision if his secure data could be transmitted in this slot and this decision is expressed as the secure transmission control vector $\zeta = (\zeta_1, \zeta_2, \dots, \zeta_N)$. The indicator variable $\zeta_n = 1$ implies that private and open messages are encoded at rate \hat{R}_n^p and $R_n^{SU} - \hat{R}_n^p$ respectively in timeslot t and $\zeta_n = 0$ means that only open messages can be transmitted at rate R_n^{SU} .

When CBS is transmitting private messages to SU n , all the other SUs except SU n are treated as potential eavesdroppers [34]. According to [46], subject to perfect private of SU n , the instantaneous private rate of SU n on subcarrier m is the achievable channel capacity minus the highest eavesdropper capacity if there is no cooperation among eavesdroppers. For each SU n , we define the most potential eavesdropper on subcarrier m as SU \tilde{n} and $\tilde{n} = \arg \max_{n', n' \neq n} a_{n'}^m$. So the security capacity of SU n on subcarrier m is:

$$\hat{R}_n^{mp} = \begin{cases} [C_n^m - \log_2(1 + \frac{p_n^m b_n^m}{1 + P_{k'}^m b_{n,P}^m})]^+ & m \in \Gamma_k^{PU}, k' \in \hat{\Gamma}_m \\ [C_n^m - \log_2(1 + p_n^m b_n^m)]^+ & m \in \Gamma_{SU} \end{cases} \quad (3)$$

where $[\cdot]^+ = \max\{\cdot, 0\}$, $b_n^m = a_n^m$ and $b_{n,P}^m$ is the cross-link CSI from PBS to SU \tilde{n} on subcarrier m . Obviously, $\hat{R}_n^{mp} = \sum_{m \in \mathbf{M}} \hat{R}_n^{mp}$. Thus the achievable private rate of user n is:

$$R_n^p = \zeta_n \hat{R}_n^{mp}$$

and the open rate of user n is: $R_n^o = R_n^{SU} - R_n^p$.

B. Queuing model

There exist data queues in both PBS and CBS. Although we want to maximize the weighted throughput of SUs, PU

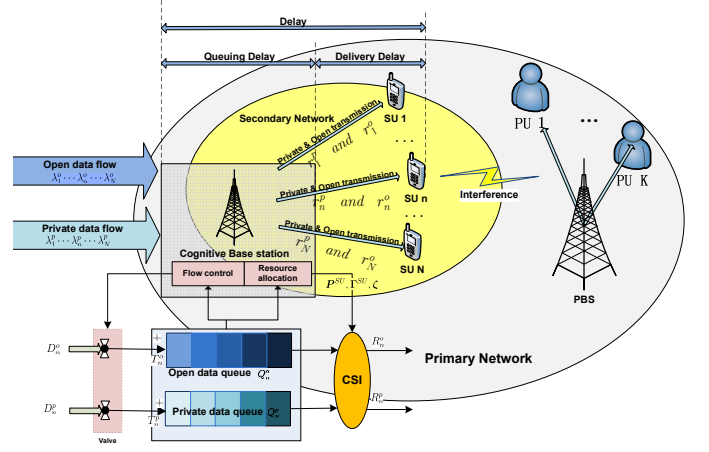


Fig. 2. Transmission model of secondary network

queue stability is a constraint in ensuring that PU's long-term throughput is not affected by SU's transmission. It is assumed that the transmission rate of PBS without interference is sufficient to serve PU's demand. However, the primary network and the secondary network will be influenced by each other if they work on the same channel. The transmission rate decrease of PU is due to the interference brought by SU transmission, while the CBS can adjust its schedule to limit interference in order to ensure that PU's time-varying rate demands can be satisfied. Later, the notation of queue stability will be used to measure whether PU's demand can be fulfilled. In [18], the interference is limited by that PU queue is kept stable under the influence caused by the only one SU access. We continue to utilize this technique in scheduling our multi-SU access system.

First, it is necessary to introduce the concept of *strong stability*. As a discrete time process, $Q(t+1) = [Q(t) - S(t)]^+ + D(t)$ is *strongly stable* if:

$$\limsup_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}\{Q(\tau)\} < \infty \quad (4)$$

In particular, a multi-queue network is stable when all queues of the network are *strongly stable*. According to *Strong Stability Theorem* in [47], for finite variable $S(t)$ and $D(t)$, strong stability implies rate stability of $Q(t)$. The definition of rate stability can be found in [47] and omitted here.

Furthermore, according to *Rate Stability Theorem* in [47], $Q(t)$ is rate stable if and only if $d \leq s$ holds where $d = \lim_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} D(\tau)$ and $s = \lim_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} S(\tau)$.

Since the data can not be delivered instantly to PUs or SUs, there are data backlogs in the PBS and CBS waiting for transmitting to respective users.

1) *PU queue*: In PBS, the data queue of PU k is updated as following:

$$Q_k(t+1) = [Q_k(t) - R_k^{PU}(t)]^+ + D_k^{PU}(t) \quad (5)$$

where $D_k^{PU}(t)$ is the amount of data packets randomly arriving at PBS during slot t with the destination of PU k . We assume $D_k^{PU}(t)$ is an i.i.d stochastic process with its upper

bound of D_k^{PU} and its long-term average arrival rates $\lambda_k = \lim_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} D_k^{PU}(\tau)$. As it has been mentioned before, Q_k should be kept stable by limiting SUs' interference to primary link. As *Rate Stability Theorem* shows, Q_k is rate stable if and only if $r_k^{PU} \geq \lambda_k$ where $r_k^{PU} \triangleq \lim_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} R_k^{PU}(\tau)$. Therefore, if PU system is strongly stable, its long-term transmission is not affected by SUs.

2) *SU data queues*: In CBS, there exist actual data queues of open and private data which are represented by Q_n^o and Q_n^p respectively for all $n \in \{1, \dots, N\}$. These queues are updated as follows:

$$Q_n^o(t+1) = [Q_n^o(t) - R_n^o(t)]^+ + T_n^o(t) \quad (6)$$

$$Q_n^p(t+1) = [Q_n^p(t) - R_n^p(t)]^+ + T_n^p(t) \quad (7)$$

All Q_k , Q_n^o and Q_n^p have initial values of zero. We define $t_n^o \triangleq \lim_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} T_n^o(\tau)$, $t_n^p \triangleq \lim_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} T_n^p(\tau)$ as the long-term time-average admission rates of open data and private data respectively. The long-term time-average service rates of Q_n^o and Q_n^p are also defined as: $r_n^o \triangleq \lim_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} R_n^o(\tau)$ and $r_n^p \triangleq \lim_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} R_n^p(\tau)$. Q_n^o and Q_n^p should be kept *strongly stable* in order to ensure the rate requirements of open and private data can be supported by the CR system, which means $t_n^o \leq r_n^o$ and $t_n^p \leq r_n^p$ hold.

Virtual queues of open data, $X_n^o(t)$, and private data $X_n^p(t)$ are introduced in (8) and (9) to assist in developing our algorithms, which would guarantee that the actual queues Q_n^o and Q_n^p are bounded deterministically in the worst case.

$$X_n^o(t+1) = [X_n^o(t) - T_n^o(t)]^+ + \mu_n^o(t) \quad (8)$$

$$X_n^p(t+1) = [X_n^p(t) - T_n^p(t)]^+ + \mu_n^p(t) \quad (9)$$

Denote μ_n^o and μ_n^p as the virtual admission rates of open data and private data, which are upper bounded by D_n^o and D_n^p respectively. Notice that X_n^o , X_n^p , μ_n^o and μ_n^p do not stand for any actual queue and data. They are only generated by the proposed algorithms. According to queuing theory, when X_n^o and X_n^p are stable, the long-term time-average value of μ_n^o and μ_n^p would satisfy:

$$\nu_n^o = \lim_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \mu_n^o(\tau) \leq t_n^o \quad (10)$$

$$\nu_n^p = \lim_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \mu_n^p(\tau) \leq t_n^p \quad (11)$$

To summarise, as shown in Fig. 2 the control space χ of the system can be expressed as $\chi = \{\mathbf{P}^{SU}, \mathbf{\Gamma}^{SU}, \zeta, \mathbf{T}\}$, which includes admission control $\mathbf{T} = \{T_n^o, T_n^p | \forall n\}$, power control decision \mathbf{P}^{SU} , subcarrier assignment $\mathbf{\Gamma}^{SU}$ and security transmission control ζ .

C. Basic constraints

1) *Power consumption constraint*: Let $E \triangleq \sum_{\forall n, \forall m} p_n^m$ as total power consumption of the whole system in one time slot. There exists a physical peak power limitation P_{max} that E cannot exceed at any time:

$$0 \leq E \leq P_{max} \quad (12)$$

The long-term time-average power consumption also has an upper bound P_{avg} , which is proposed for energy conservation:

$$e \leq P_{avg} \quad (13)$$

where $e = \lim_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}\{E(\tau)\}$

2) *Delay-limited model*: The queuing delay is defined as the time a packet waits in a queue until it can be transmitted. Each SU has a long-term time-average queuing delay ρ_n^o for its open data transmission. To each SU, it proposes a delay constraint ρ_n as in (14) for its open transmission.

$$\rho_n^o \leq \rho_n \quad (14)$$

IV. PROBLEM FORMULATION

Considering the simplicity and understandability of mathematical analysis, a special case of one single primary link is considered in the following. In the single PU case, the only one PU is indexed with number 0. In part C of Section V, the general results of multi-PU case are listed for completeness.

A. Optimization objective and constraints

Following above descriptions, the objective of this paper is to improve throughput of secondary network while ensuring stability of primary network. So the problem is formulated as: Maximize the sum weighted admission rates of all SUs and stabilize the PU data queue Q_0 at the same time. Let θ_n and φ_n for all n be the nonnegative weights for private and open data throughput. Then the optimal problem can be formulated as:

$$\begin{aligned} & \text{Maximize} && \sum_{n=1}^N \{\theta_n t_n^p + \varphi_n t_n^o\} \\ & \text{Subject to:} && 0 \leq t_n^p \leq \lambda_n^p, \forall n \\ & && 0 \leq t_n^o \leq \lambda_n^o, \forall n \\ & && \mathbf{t} = (t_n^p, t_n^o) \in \Upsilon \\ & && \limsup_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}\{Q_0(\tau)\} < \infty \end{aligned} \quad (15)$$

where Υ is the *network capacity region* of secondary links. Define the service rate vector as $\mathbf{v} = (r_n^o, r_n^p)$. The definition of *network capacity region* Υ is the region of all non-negative service rate vectors \mathbf{v} for any possible control actions [47]. When the CBS takes a kind of control policy under a certain channel condition, the secondary links will have a decided network capacity and the *network capacity region* is the set of network capacities under all possible control policies and all channel conditions. In the proposed system, the control policy of CBS should fulfill subcarrier assignment rule (1), peak power constraint (12) and stabilize all queues including actual queues and virtual queues. So actually, the control policy that can achieve the network capacity region should

satisfy the following constraints:

$$\begin{cases} (1), (12) \\ \limsup_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}\{Q_n^o(\tau)\} < \infty \\ \limsup_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}\{Q_n^p(\tau)\} < \infty \\ \limsup_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}\{X_n^o(\tau)\} < \infty \\ \limsup_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}\{X_n^p(\tau)\} < \infty \end{cases}$$

Theoretically, we can get the optimal solution to (15) if we get the distribution of the system CSI and external data arrival rate beforehand. However, this information can not be obtained accurately. In this paper an online algorithm requiring only current information of queue state and channel state is proposed and will be described in detail then.

B. Optimality of SU overlay

Before detailing the control algorithm, it should be specified the conditions that make SU overlay play a positive role in this cognitive transmission model other than traditional access methods. We focus on presenting a sufficient condition on overlay for constant channel conditions here, then we will extend it to time-varying situation.

In the case of static network condition, the optimal problem of SUs' weighted throughput is simplified as

$$\begin{aligned} \text{Maximize: } & \sum_{n=1}^N \{\theta_n r_n^p + \varphi_n r_n^o\} \\ \text{Subject to } & r_0^{PU} = \lambda_0 \end{aligned} \quad (16)$$

where we only consider the optimal case when $r_0^{PU} = \lambda_0$. Notice here, the system maximal weighted sum data rate under full overlay scheme must be greater than or at least no worse than that when SU can only access the subcarrier which is not occupied by PU. It is easy to understand that full overlay is a more general access scheme than spectrum overlay which is a special access situation. We can get an intuition that when all subcarriers are assumed to be accessed by PU, SU data rate would be positive under full overlay scheme instead of zero under traditional overlay scheme. Thus what we want to prove is the sufficient condition of that SUs perform better in consideration of PU transmission other than accessing the licensed subcarrier roughly. Let κ be the fraction of time that PU is actively transmitting, thus:

$$r_0^{PU} = \kappa \sum_{m \in \Gamma_0^{PU}} \log_2 \left(1 + \frac{A_0^m P_0^m}{1 + a_{0S}^m p_n^m} \right) \quad (17)$$

$$\begin{aligned} r_n^p = \{ & \sum_{m \in \Gamma_0^{PU}} \{(1 - \kappa)[\log_2(1 + a_n^m p_n^m) - \log_2(1 + b_n^m p_n^m)]^+ \\ & + \kappa[\log_2(1 + \frac{p_n^m a_n^m}{1 + P_0^m a_{nP}^m}) - \log_2(1 + \frac{p_n^m b_n^m}{1 + P_0^m b_{nP}^m})]^+ \} \\ & + \sum_{m \in \Gamma_{SU}} [\log_2(1 + a_n^m p_n^m) - \log_2(1 + b_n^m p_n^m)]^+ \} \zeta_n \quad (18) \end{aligned}$$

$$\begin{aligned} r_n^o = & \sum_{m \in \Gamma_{SU}} \log_2(1 + a_n^m p_n^m) + \sum_{m \in \Gamma_0^{PU}} [\kappa \log_2(1 + \\ & \frac{p_n^m a_n^m}{1 + P_0^m a_{nP}^m}) + (1 - \kappa) \log_2(1 + a_n^m p_n^m)] - r_n^p \quad (19) \end{aligned}$$

We have the following lemma:

Lemma 1: In high SINR region, a sufficient condition for full overlay to be optimum in SU n accessing subcarrier m (both security and open transmission) is:

$$a_{0S}^m \leq \min\{C_{nm}^1, C_{nm}^2\}, \forall m \quad (20)$$

where $C_{nm}^1 = b_n^m / [(1 + P_0^m b_{nP}^m + b_n^m P_{max}) \log_2(1 + b_n^m P_{max})]$, $C_{nm}^2 = a_n^m / [(1 + P_0^m a_{nP}^m + a_n^m P_{max}) \log_2(1 + a_n^m P_{max})]$.

We can have an intuitive explanation on *Lemma 1*, for SU n 's accessing subcarrier m . If the cross link (from CBS to primary link) condition is bad enough (worse than weighted CBS-to-SU channel condition C_{nm}^2 and weighted CBS-to-eavesdropper channel condition C_{nm}^1), the full overlay scheme would be the optimal scheme when both security and open transmission happen. The proof of *Lemma 1* can be found in Appendix C.

It would be obvious to derive the following lemma on sufficient condition of optimality of the whole system overlay. Thus we get:

Lemma 2: In high SINR region, a sufficient condition for full overlay to be optimum in the whole OFDMA-based CR system is:

$$a_{0S}^m \leq \min_n \{C_{nm}^1, C_{nm}^2\}, \forall m \quad (21)$$

Notice that, the sufficient condition does not mean that subcarrier $m \in \Gamma_0^{PU}$ would provide a greater data rate than $m' \in \Gamma_{SU}$ under the same power allocation scheme. It means that for $m \in \Gamma_0^{PU}$, full overlay would achieve the optimal result other than any other access policy such as partial overlay or underlay. We assume the sufficient condition of *Lemma 2* is fulfilled in this paper and we proceed considering time-varying channels then.

V. ONLINE CONTROL ALGORITHM

It is worth noticing that problem (15) has long-term time-average limitations on power consumption and queuing delay. Using the technique similar to [47], we construct power virtual queue Y and delay virtual queue Z_n to track the power consumption and queuing delay respectively. These virtual queues do not exist in practice, and they are just generated by the iterations of (22) and (23):

$$Y(t+1) = [Y(t) - P_{avg}]^+ + E(t) \quad (22)$$

$$Z_n(t+1) = [Z_n(t) - \rho_n \mu_n^o]^+ + Q_n^o(t) \quad (23)$$

Similar to actual queues, Y and Z_n have initial values of zero. According to *Necessary Condition for Rate Stability* in [47], if Y is stable, constraint (13) is satisfied. In addition, if Z_n is stable, $q_n^o = \lim_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}\{Q_n^o(\tau)\} \leq \rho_n \nu_n \leq \rho_n t_n^o$ holds. According to Little's Theorem, $q_n^o / t_n^o = \rho_n^o$, when Z_n is stable, the delay constraint (14) would be achieved. It will be proven that the proposed optimal control algorithm can stabilize these queues in section VI, that is to say the long-term time-average constraints are fulfilled.

Using virtual queues X_n , Z_n and Y , we decouple problem (15) into two parts: one is *flow control algorithm* which decides the admission of data, and another is *resource allocation algorithm* in charge of subcarrier assignment, power

allocation and secure transmission control in every slot. All these control actions aim at secondary links and happen in CBS. The whole algorithm is named CBS-side online control algorithm (COCA).

A. Flow control algorithm

When external data arrives at CBS, CBS will decide whether to admit it according to queue lengths. Let V be a fixed non-negative control parameter. Let $q_{max}^o \geq \mu_{max}$ and $q_{max}^p \geq D_{max}$ hold. They are actually the deterministic worst case upper bounds of relative queue length to be proven later. The flow control rules of open data and private data are obtained by solving (24) and (25) respectively:

$$\begin{aligned} \text{Minimize} \quad & T_n^o [Q_n^o - q_{max}^o + \mu_{max}] \\ \text{Subject to:} \quad & 0 \leq T_n^o \leq D_n^o \end{aligned} \quad (24)$$

$$\begin{aligned} \text{Minimize} \quad & T_n^p [Q_n^p - q_{max}^p + D_{max}] \\ \text{Subject to:} \quad & 0 \leq T_n^p \leq D_n^p \end{aligned} \quad (25)$$

The corresponding solutions to (24) and (25) are easy to get:

$$T_n^o = \begin{cases} 0 & \text{if } Q_n^o - q_{max}^o + \mu_{max} \geq 0 \\ D_n^o & \text{otherwise} \end{cases} \quad (26)$$

$$T_n^p = \begin{cases} 0 & \text{if } Q_n^p - q_{max}^p + D_{max} \geq 0 \\ D_n^p & \text{otherwise} \end{cases} \quad (27)$$

Here we can have an intuitive explanation on flow control rules. They work like valves. When any actual data queue exceeds some threshold, the corresponding valve would turn off and no data would be admitted.

As to virtual variable μ_n^o and μ_n^p , there are also their respective virtual flow control algorithms (28) and (29) so as to update virtual queues X_n^o and X_n^p which will play an important role in resource allocation:

$$\begin{aligned} \text{Minimize} \quad & \mu_n^o \left[\frac{q_{max}^o - \mu_{max}}{q_{max}^o} X_n^o - \rho_n Z_n - V \varphi_n \right] \\ \text{Subject to:} \quad & 0 \leq \mu_n^o \leq D_n^o \end{aligned} \quad (28)$$

$$\begin{aligned} \text{Minimize} \quad & \mu_n^p \left[\frac{q_{max}^p - D_{max}}{q_{max}^p} X_n^p - V \theta_n \right] \\ \text{Subject to:} \quad & 0 \leq \mu_n^p \leq D_n^p \end{aligned} \quad (29)$$

Solutions to (28) and (29) are (30) and (31) respectively:

$$\mu_n^o = \begin{cases} 0 & \text{if } \left(\frac{q_{max}^o - \mu_{max}}{q_{max}^o} X_n^o - \rho_n Z_n - V \varphi_n \right) \geq 0 \\ D_n^o & \text{otherwise} \end{cases} \quad (30)$$

$$\mu_n^p = \begin{cases} 0 & \text{if } \left(\frac{q_{max}^p - D_{max}}{q_{max}^p} X_n^p - V \theta_n \right) \geq 0 \\ D_n^p & \text{otherwise} \end{cases} \quad (31)$$

B. Resource allocation algorithm

The resource allocation policy can be found in solving the following optimization problem.

$$\begin{aligned} \text{Maximize} \quad & U(\mathbf{P}^{SU}, \zeta) \\ \text{Subject to:} \quad & (1), (12) \end{aligned} \quad (32)$$

$$\text{where } U(\mathbf{P}^{SU}, \zeta) = \sum_{n=1}^N \left(\frac{X_n^o Q_n^o}{q_{max}^o} R_n^o + \frac{X_n^p Q_n^p}{q_{max}^p} R_n^p \right) + Q_0 R_0^{PU} - YE.$$

At the beginning of every slot, all $X_n^o, X_n^p, Q_n^o, Q_n^p$ and Y can be regarded as constants because they all have been decided in the previous slot. Q_0 can be estimated by CBS by overhearing PBS feedback. In section VII we propose an imperfect estimation scheme of Q_0 and compare the performances of perfect and imperfect estimations in simulations. Notice that, the resource allocation is determined at the beginning of every slot and all queues are updated at the end of every slot.

Firstly, we can easily decide the vector ζ maximizing U by assuming that all elements of ζ are continuous variables between 0 and 1 and in further discussion, we can get a discrete implementation of ζ_n .

We take partial derivative in $U(\mathbf{P}^{SU}, \zeta)$ with respect to ζ_n :

$$\frac{\partial U(\mathbf{P}^{SU}, \zeta)}{\partial \zeta_n} = \left(\frac{X_n^p Q_n^p}{q_{max}^p} - \frac{X_n^o Q_n^o}{q_{max}^o} \right) \sum_{m=1}^M \hat{R}_n^{mp} \quad (33)$$

Observing (33), $\sum_{m=1}^M \hat{R}_n^{mp}$ is no-negative and U is monotonic in ζ_n , and thus the optimality condition of secure transmission control is:

$$\zeta_n^* = \begin{cases} 1 & \text{if } \left(\frac{X_n^p Q_n^p}{q_{max}^p} - \frac{X_n^o Q_n^o}{q_{max}^o} \right) \geq 0 \\ 0 & \text{otherwise} \end{cases} \quad (34)$$

Then we use ζ^* to assign subcarrier and power which is the solution to the following optimization problem PS,

$$\begin{aligned} \text{PS:} \quad & \text{Maximize } \tilde{U}(\mathbf{P}^{SU}) \\ & \text{Subject to: } (1), (12) \end{aligned}$$

where $\tilde{U}(\mathbf{P}^{SU}) = U(\mathbf{P}^{SU}, \zeta^*)$. PS is a typical Weighted Sum Rate (WSR) maximization problem, and it is difficult to find a global optimum since $\tilde{U}(\mathbf{P}^{SU})$ is neither convex nor concave of \mathbf{P}^{SU} . Obviously, PS has a typical D.C. structure which can be optimally solved by D.C. programming [48]. In [49] there lists a dual decomposition iterative suboptimal algorithm solving this kind of constrained nonconvex problem instead of D.C. programming. In addition, because of the characteristics of OFDMA networks, the duality gap is equal to zero even if PS is nonconvex when the number of subcarriers is close to infinity [50]. So we take a more computationally effective dual method to solve PS and due to space limitation, we give the key steps here only.

We define $R_n^{mp} = \zeta_n^* \hat{R}_n^{mp}$ and $R_n^{mo} = C_n^m - R_n^{mp}$. Then the Lagrange function of PS is expressed as:

$$\begin{aligned} J(\delta, \mathbf{P}^{SU}) = & \sum_{m=1}^M \left\{ \sum_{n=1}^N \left[\frac{X_n^o Q_n^o}{q_{max}^o} R_n^{mo} + \frac{X_n^p Q_n^p}{q_{max}^p} R_n^{mp} - Y p_n^m \right] \right. \\ & \left. + Q_0 R_0^m \right\} + \delta (P_{max} - E) \end{aligned} \quad (35)$$

where δ is the non-negative Lagrange multiplier for the peak power constraint in problem PS. The dual problem of PS is: $\min_{\delta \geq 0} H(\delta)$, where $H(\delta) = \max_{\mathbf{P}^{SU} \geq 0} \{J(\delta, \mathbf{P}^{SU})\}$.

When δ is fixed, we can decide the parameters \mathbf{P}^{SU} maximizing the objective of $H(\delta)$. Observing $H(\delta)$, we find that it can be decoupled into M subproblem as:

$$\begin{aligned} H(\delta) &= \sum_{m=1}^M \max_{\mathbf{P}^{SU}} J_m(\delta, \mathbf{P}^{SU}) + \delta P_{max} \\ &= \sum_{m=1}^M \max_{\mathbf{P}^{SU}} \sum_{n=1}^N J_n^m(\delta, p_n^m) + \delta P_{max} \end{aligned} \quad (36)$$

where $\mathbf{P}_{SU}^m = \{p_n^m | 1 \leq n \leq N\}$, $J_m(\delta, \mathbf{P}_{SU}^m) = \sum_{n=1}^M J_n^m(\delta, p_n^m)$, $J_n^m(\delta, p_n^m) = \frac{X_n^o Q_n^o}{q_{max}^o} R_n^{mo} + \frac{X_n^p Q_n^p}{q_{max}^p} R_n^{mp} - (Y + \delta)p_n^m + Q_0 R_0^m(n)$ and

$$R_0^m(n) = \begin{cases} 0 & m \in \Gamma_{SU} \\ R_0^m & m \in \Gamma_0^{PU}, \varpi_n^m = 1 \end{cases} \quad (37)$$

For $m \in \Gamma_{SU}$, we can get p_n^{m*} by taking partial derivative of $J(\delta, \mathbf{P}^{SU})$ with respect to p_n^m and making (38) equal to zero:

for $m \in \Gamma_{SU}$:

$$\begin{aligned} \frac{\partial(J(\delta, \mathbf{P}^{SU}))}{\partial p_n^m} &= \frac{X_n^o Q_n^o}{q_{max}^o} \frac{1}{\ln 2} \left\{ \frac{a_n^m}{1 + p_n^m a_n^m} - \zeta_n \left[\frac{a_n^m}{1 + p_n^m a_n^m} \right. \right. \\ &\quad \left. \left. - \frac{b_n^m}{1 + p_n^m b_n^m} \right] \right\} + \frac{X_n^p Q_n^p}{q_{max}^p} \frac{1}{\ln 2} \zeta_n \left[\frac{a_n^m}{1 + p_n^m a_n^m} - \frac{b_n^m}{1 + p_n^m b_n^m} \right] \\ &\quad - (Y + \delta) \end{aligned} \quad (38)$$

However, for $m \in \Gamma_0^{PU}$, a global optimal solution p_n^{m*} maximizing J_n^m can be got easily by an exhaustive search such as clustering methods or enumerative methods [51] and it is computationally tractable [50], [52].

Substituting (34) and p_n^{m*} into $J_n^m(\delta, \mathbf{P}^{SU})$, the results are denoted as J_n^{m*} . For any subcarrier m , it will be assigned to the user who has the biggest $J_n^{m*}(\delta, \mathbf{P}^{SU})$. Let n_m^* be the result of subcarrier m 's assignment which is given by:

$$n_m^* = \arg \max_n J_n^m, \forall n \quad \text{and} \quad \varpi_n^{m*} = \begin{cases} 1 & \text{if } n = n_m^* \\ 0 & \text{otherwise} \end{cases} \quad (39)$$

Let $E^* = \sum_{n=1}^N \sum_{m=1}^M p_n^{m*} \varpi_n^{m*}$. As to the value of δ , we use subgradient method to update it as in (40),

$$\delta(i+1) = [\delta(i) - \varsigma \Delta\delta(i)]^+ \quad (40)$$

where $\Delta\delta(i) = P_{max} - E^*(t, i)$. $\Delta\delta(i)$ is the subgradient of $H(\delta)$ at δ and ς is the step size which should be a small positive constant. In addition, index i stands for iteration number. When the subgradient method converges, the resource allocation is completed.

From the above description, we can find some principles of resource allocation.

Remark 1: In (34), both virtual and actual queues of open as well as private data reflect the gap between the corresponding user's demand on data rate and the data rate that the system can provide. Thus, $\frac{X_n^o Q_n^o}{q_{max}^o}$ and $\frac{X_n^p Q_n^p}{q_{max}^p}$ can be regarded as the transmission urgency of open data and private data. Only when the transmission urgency of private data exceeds open data, CBS would allocate some resource to transmit private data. Otherwise, CBS would use the user's entire resource to

TABLE I
ALGORITHM DESCRIPTIONS

Proposed online control algorithm in timeslot t
1) Flow control: Use (26), (27), (30), (31) to calculate T_n^o , T_n^p , μ_n^o and μ_n^p respectively.
2) Resource allocation: a) Set the Lagrange multiplier $\delta = \delta_{ini}$: (δ_{ini} : An initial value of δ). b) For each (n, m) i) Use (34) to calculate ζ_n^* . ii) Use (38) or exhaustive search to find p_n^{m*} . iii) Use (39) to calculate ϖ_n^{m*} . c) Use (40) to update δ and calculate $\Delta\delta(i)$. d) If $ \Delta\delta(i) > \Delta\delta_c$, goto b) , else proceed. ($\Delta\delta_c$: converge condition of $\Delta\delta$)
3) Update the queues: Use (6), (7), (8), (9), (22) and (23) to update all queues including Q_n^o , Q_n^p , X_n^o , X_n^p , Y , Z_n .

transmit open data due to delay constraint. In PS, it is easy to find that a bigger Y results in less power allocated to every user, which will reduce the system power consumption. Also we let Q_0 to be the weights of R_0^{PU} in PS. It means that if the transmission pressure of PU is high, CBS will allocate less power in subcarrier set Γ_0^{PU} to avoid causing too much interference on primary link.

Remark 2: In the sub-problem of PS, the transmission power of PBS is assumed to be external variables. Even for the worst case that PBS does not control its transmission power actively, the proposed resource allocation algorithm aims to maximize $Q_0 R_0^{PU}$ in PS by adjusting the interference from the secondary networks to primary networks. Thus, it can be found that the proposed algorithm actually does not affect the energy consumption of primary networks too much.

C. Control algorithm of Multi-PU case

Flow control algorithm is the same as (26), (27), (30) and (31).

Resource allocation of multi-PU implementation is the solution to problem MPS:

MPS:

Maximize:

$$\sum_{n=1}^N \frac{X_n^o Q_n^o}{q_{max}^o} R_n^o + \sum_{n=1}^N \frac{X_n^p Q_n^p}{q_{max}^p} R_n^p + \sum_{k=1}^K Q_k R_k^{PU} - Y E \quad (41)$$

Subject to: (1), (12)

In next section, the algorithm performance with single PU is analysed. It is easy for readers to prove that multi-PU implementation ensures primary data queue stability and furthermore enjoys a similar performance as single PU situation.

VI. ALGORITHM PERFORMANCE

Before the analysis it is necessary to introduce some auxiliary variables. Let $\mathbf{t}^* = (t_n^{p*}, t_n^{o*})$ be the solution to the

following problem:

$$\begin{aligned} \max_{\mathbf{t}: \mathbf{t} \in \Upsilon} \quad & \sum_{n=1}^N \theta_n t_n^p + \varphi_n t_n^o, \\ \text{Subject to:} \quad & e \leq P_{avg} \end{aligned}$$

And $\mathbf{t}^*(\epsilon) = (t_n^{p,*}(\epsilon), t_n^{o,*}(\epsilon))$ denotes the solution of:

$$\begin{aligned} \max_{\mathbf{t}: \mathbf{t} + \epsilon \in \Upsilon} \quad & \sum_{n=1}^N \theta_n t_n^p + \varphi_n t_n^o \\ \text{Subject to:} \quad & e \leq P_{avg} \end{aligned}$$

According to [53], it is true that:

$$\lim_{\epsilon \rightarrow 0} \sum_{n=1}^N \{\theta_n t_n^{p,*}(\epsilon) + \varphi_n t_n^{o,*}(\epsilon)\} = \sum_{n=1}^N \{\theta_n t_n^{p,*} + \varphi_n t_n^{o,*}\} \quad (42)$$

The algorithm performance will be listed in *Theorem 1* and *Theorem 2*.

Theorem 1: Employing the proposed algorithm, both actual queues of open data $Q_n^o(t)$ and private data $Q_n^p(t)$ in CBS have deterministic worst-case bounds:

$$Q_n^o(t) \leq q_{max}^o, Q_n^p(t) \leq q_{max}^p, \forall t, \forall n \quad (43)$$

Theorem 2: Given

$$q_{max}^o > \mu_{max} + \frac{C_{max}^{o^2} + \mu_{max}^2}{2\epsilon}, \quad (44)$$

$$q_{max}^p \geq D_{max} + \frac{C_{max}^{p^2} + D_{max}^2}{2\epsilon}, \quad (45)$$

$$\rho_n > \frac{q_{max}^o}{\nu_n^{o,*}(\epsilon)}, \forall n \quad (46)$$

where ϵ is positive and can be chosen arbitrarily close to zero. The proposed algorithm performance is bounded by:

$$\begin{aligned} \liminf_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \sum_{n=1}^N \{\theta_n T_n^p(\tau) + \varphi_n T_n^o(\tau)\} \\ \geq \sum_{n=1}^N \{\varphi_n t_n^{o,*}(\epsilon) + \theta_n t_n^{p,*}(\epsilon)\} - \frac{B}{V} \end{aligned} \quad (47)$$

where B is a positive constant independent of V and its expression can be found in appendix B.

In addition, the algorithm also ensures that the long-term time-average sum of PU queue Q_0 and virtual queues X_n^o , X_n^p , Z_n , Y has an upper bound:

$$\begin{aligned} \limsup_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \sum_{n=1}^N (X_n^o + X_n^p + Z_n) + Y + Q_0 \\ \leq \frac{B + V \sum_{n=1}^N \{\theta_n t_n^{p,*} + \varphi_n t_n^{o,*}\}}{\sigma} \end{aligned} \quad (48)$$

where $0 \leq \sigma \leq \epsilon$. The proof of Theorem 1 is in appendix A. Theorem 2 and the definition of σ can be found in appendix B.

Remark 3 (Network stability): According to the definition of *strongly stability* as shown in (4), (43) and (48) indicate the stabilities of all queues in the network system. As a result, the network system is stabilized and the long-term time-average constraints of delay and power are satisfied. Notice

here that Q_0 's stability is proved means the PU queue stability constraint is fulfilled. Q_0 's stability means that the long-term throughput performance is uninfluenced. In addition, if PU's arrival rates are within the stability region of PU networks, Q_0 's stability can be ensured by the proposed scheduling algorithm for any transmission power of PU base station. Therefore, the transmission power of PU network is not affected in this situation. Furthermore, (43) states that all the actual queues of open data and private data have deterministic upper bounds, and this characteristic means that the CBS can accommodate the random arrival packets with finite buffer.

Remark 4 (Optimal throughput performance): (47) states a lower-bound on the weighted throughput that our algorithm can achieve. Since B is a constant independent of V , our algorithm would achieve a weighted throughput arbitrarily close to $\sum_{n=1}^N \{\varphi_n t_n^{o,*}(\epsilon) + \theta_n t_n^{p,*}(\epsilon)\}$ for some $\epsilon \geq 0$. Furthermore, given any $\epsilon \geq 0$, we can get a better algorithm performance by choosing a larger V without improving the buffer sizes. In addition, as it is shown in (42), when ϵ tends to zero, our algorithm would achieve a weighted throughput arbitrarily close to $\sum_{n=1}^N \{\varphi_n t_n^{o,*} + \theta_n t_n^{p,*}\}$ with a tradeoff in queue length bounds and long-term time-average delay constraints as shown in (44)-(46). Thus we can see that with some certain finite buffer sizes, the proposed algorithm can provide arbitrarily-close-to-optimal performance by choosing V , and V 's influence on queue length is shifted from actual queues to virtual queues.

VII. IMPLEMENTATION WITH IMPERFECT ESTIMATION

CBS needs the information of queue length from primary networks to decide the resource allocation among SUs. [17] considers a situation that queue length information is shared among all the nodes, but in CR environment it is impossible to know the non-cooperative PU's queue information precisely. Compared with getting perfect information about Q_k , it is more realistic to know the time-average packet arrival rate of PUs. Considering this, in this section, we propose an imperfect estimation of Q_k by CBS. And the performance of this estimation will be showed in simulation section. If the PU k is busy, the estimated queue length in CBS is:

$$\hat{Q}_k(t+1) = [\hat{Q}_k(t) - R_k^{PU}(t)]^+ + (\lambda_k + \iota) \quad (49)$$

where ι is an over-estimated slack variable to promise primary link stability. CBS can get the precise information when PU is idle by listening to primary link ACK to find that no power is used to transmit PU k 's data packets. In this situation, $\hat{Q}_k = Q_k = 0$ perfectly holds.

As to the control algorithm, we use \hat{Q}_k to substitute Q_k in resource allocation algorithm. For simplicity, we name this implementation COCA-E (CBS-side online control algorithm with estimated PU queue).

VIII. SIMULATION

In this section, we firstly simulate COCA performance in an exemplary CR system with a single primary link and secondary network consisting of one CBS, eight SUs and 64 subcarriers. All weights of open data and private data

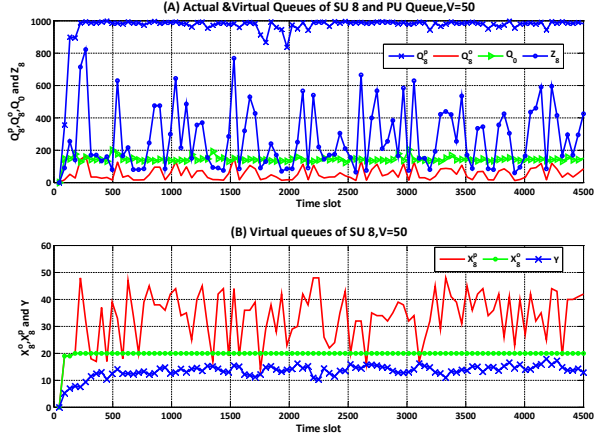


Fig. 3. Queue evolutions over 4500 slots

are set to be 0.8 and 1 respectively. The main algorithm parameters of secondary network are set as: $P_{avg} = 0.8W$, $P_{max} = 1W$, $\rho_n = 60, \forall n$, $q_{max}^o = 200$, $q_{max}^p = 1000$, $\mu_{max} = 50$, $D_{max} = 20$ and $\lambda_n^o = n * 0.1 * D_{max}$, $\lambda_n^p = n * 0.1 * \mu_{max}$, for $n \in \{1, 2, \dots, 8\}$. The long-term time-average arrival rate of PU λ_0 is set to be 140 and $D_{max}^{PU} = 200$. We simulate the multipath channel of primary and secondary networks as Rayleigh fading channels and the shadowing effect variances are 10 dB. The cross-link channels between PBS to SUs and CBS to PU are simulated as long-scale fading. All parameters in the following parts are set the same as these mentioned here, except for other specification.

In Fig. 3 and Fig. 4, we set average value of a_{0S}^m , $\overline{a_{0S}} = 0.35$, and average value of a_{nP}^m , $\overline{a_{nP}} = 21$, $V = 50$ and we show both primary and secondary networks' queue evolution over 4500 slots. Because all SUs' data queues (Q_n^o, Q_n^p) and virtual queues (X_n^o, X_n^p, Z_n) enjoy similar trends, we take SU 8 as an example. Fig. 3 (A) shows the dynamics of SU 8's data queues Q_8^o, Q_8^p , virtual delay queue Z_8 and PU queue Q_0 . It is observed that both actual data queues are strictly lower than their own deterministic worst case upper bound, which verifies *Theorem 1*. That Q_0 is stable in Fig. 3 (A) illustrates that our algorithm can ensure PU queues stability from simulation aspect. Besides, in Fig. 3(B), we can also see that virtual queues X_8^o, X_8^p and Y are bounded. So Fig. 3 shows that all queues are bounded, which means that the network system is stabilized and the long-term time-average constraints of delay and power are satisfied.

Fig. 4 directly shows eight SUs' long-term time-average admitted rates and service rates of open data and private data, respectively. Notice that, every user's admitted rate is smaller than service rate and this promises the stabilities of actual data queues.

Fig. 5 shows the relationship between the weighting parameters and long-term time-average service rates. To show the effects more clearly, we consider the scenario consisting of only one SU and one PU with fixed $\varphi_1 = 450$ and variational $\theta_1 \in \{0, 90, 180, 270, 360, 450\}$. The long-term time-average arrival rate of SU is set as: $\lambda_1^p = 8$ and $\lambda_1^o = 260$. The control

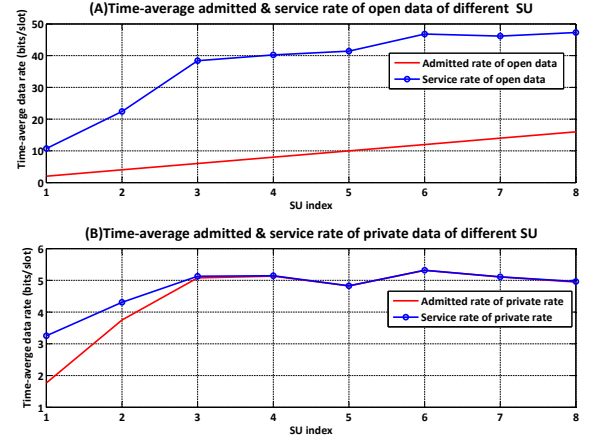
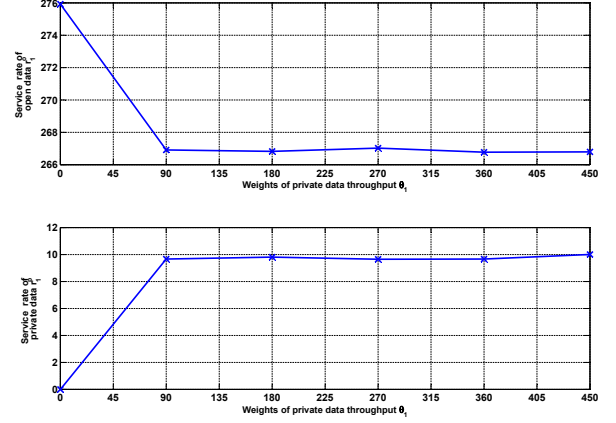


Fig. 4. Long-term time-average admitted and service rate of all SUs

Fig. 5. The time average service rate (r_1^p and r_1^o) versus the weights of private data θ_1

parameter V is set to be 380. Each value in Fig. 5 is obtained by averaging the converged results of 5000 times. Fig. 5 shows with the increase of θ_1 the long-term time-average service rate of private data increases while the one of open data decreases, which illustrates the effect of throughput weights on long-term time-average service rates.

Fig. 6 demonstrates the relationship between different long-term time-average network performance versus control parameter V . In order to compare PU and SU performance, the similar scenario including one PU and one SU is also considered here. The average data arrival rates of SU are set as: $\lambda_1^o = 250$ and $\lambda_1^p = 10$. In general, the bigger V results in the higher SU open and private transmission rates as Fig. 6 (B) and Fig. 6 (C) respectively show. Fig. 6 (A) demonstrates PU transmission rate decreases as V increases. Notice here, although r_0^{PU} decreases, even when $V = 380$, r_0^{PU} approximates 146 and is greater than $\lambda_0 = 140$, which preserves PU queue stability. Fig. 6 (D) shows the queuing delay performance also improves as V increases.

The implementation of COCA-E with imperfect estimated Q_0 is simulated. We set the over-estimated slack variable

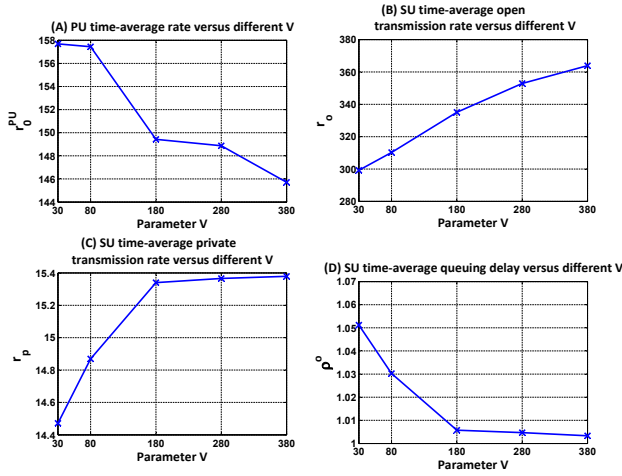


Fig. 6. COCA performances (long-term time-average PU rate, SU rates and SU queuing delay) versus control parameter V .

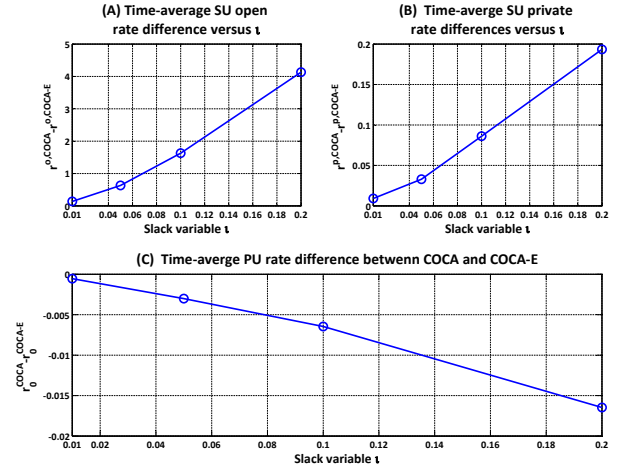


Fig. 8. The long-term time-average rate differences of COCA and COCA-E implementation versus different over-estimated slack variable ι .

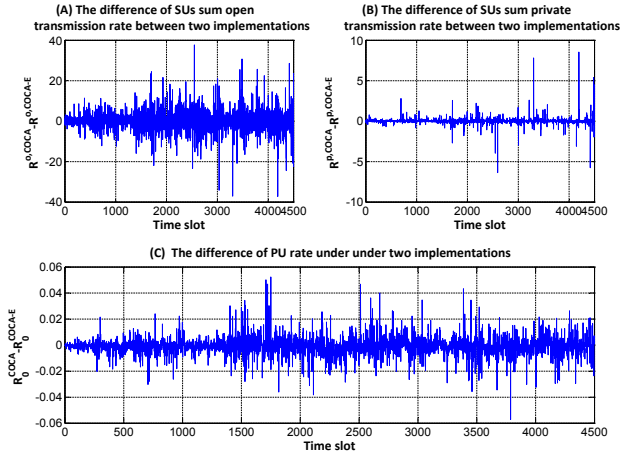


Fig. 7. The rate difference of COCA and COCA-E implementation during 4500 slots.

ι to be 0.01. We show the differences of the sum service rate of SUs and R_0^{PU} between COCA and COCA-E in Fig. 7 (A), Fig. 7 (B) and Fig. 7 (C), respectively. We can see that all the differences are around zero, and SU sum rate is more effected than R_0^{PU} by the imperfect estimation of PU queue information. More directly, the influence of ι on the long-term time-average rate difference between COCA and COCA-E is simulated in Fig. 8, where each record is an averaged result of 1000 converged results. Fig. 8 (C) shows that $r_0^{COCA} - r_0^{COCA-E}$ becomes more negative as ι increases, which means that the rate decline of PU caused by SU transmissions decreases as ι increases. More directly, if we want to make sure PU transmission is less influenced, we should choose a larger ι . While a larger ι inevitably makes SUs' transmission rates decrease as Fig. 8 (A) and Fig. 8 (B) show.

IX. CONCLUSIONS

In this paper, we propose a cross-layer scheduling and dynamic spectrum access algorithm for maximizing the long-

term average throughput of open and private information in an OFDMA-based CR network. We derive the sufficient condition to guarantee that full overlay is optimal in this system. The proposed algorithm can provide a flexible scheduling implementation of open and private information while ensuring the stability of primary networks as well as performance requirements in CR systems with finite buffer size. Furthermore, the proposed algorithm is proved to be close to optimality with current network states in time-varying environments.

APPENDIX A PROOF OF THEOREM 1

Supposing there exists a slot t satisfying $Q_n^o(t) \leq q_{max}^o$, it is obviously true for all queues initialized to zero. We prove that for $t+1$ the same holds. Obviously, there exists two cases. Firstly, we suppose $Q_n^o(t) \leq q_{max}^o - \mu_{max}$ and we can easily get $Q_n^o(t+1) \leq q_{max}^o$. Else, if $Q_n^o(t) > q_{max}^o - \mu_{max}$, then according to (26), $T_n^o(t) = 0$. Then

$$Q_n^o(t+1) \leq Q_n^o(t) \leq q_{max}^o.$$

The proof of $Q_n^p \leq q_{max}^p$ is similar and omitted here.

APPENDIX B PROOF OF THEOREM 2

Let $\mathbf{Q} = \{Q_0, Q_n^o, Q_n^p, X_n^o, X_n^p, Y, Z_n\}$. We define Lyapunov function $L(\mathbf{Q})$ as:

$$L(\mathbf{Q}) = \frac{1}{2} \left\{ \sum_{n=1}^N \left[\frac{q_{max}^o - \mu_{max}}{q_{max}^o} X_n^{o2} + Z_n^2 + \frac{1}{q_{max}^o} Q_n^{o2}(t) X_n^o + \frac{q_{max}^p - D_{max}}{q_{max}^p} X_n^{p2} + \frac{1}{q_{max}^p} Q_n^{p2} X_n^p \right] + Y^2 + Q_0^2 \right\} \quad (50)$$

According to [47], $\Delta L(\mathbf{Q})$ is defined as the conditional Lyapunov drift for slot t :

$$\Delta L(\mathbf{Q}) \triangleq \mathbb{E}\{L(\mathbf{Q}(t+1)) - L(\mathbf{Q}(t)) | \mathbf{Q}(t)\} \quad (51)$$

According to $(|x - y| + z)^2 \leq x^2 + y^2 + z^2 - 2x(y - z)$, we can get the results below:

$$\frac{q_{max}^o - \mu_{max}}{q_{max}^o} [X_n^{o2}(t+1) - X_n^{o2}(t)] \leq \frac{q_{max}^o - \mu_{max}}{q_{max}^o} \{2\mu_{max}^2 - 2X_n^o(t)[T_n^o(t) - \mu_n^o(t)]\} \quad (52)$$

$$\frac{[Q_n^{o2}(t+1)X_n^o(t+1) - Q_n^{o2}(t)X_n^o(t)]}{q_{max}^o} \leq q_{max}^o \mu_{max} + \frac{(\mu_{max}^2 + C_{max}^{o2}) - 2Q_n^o(t)[R_n^o(t) - T_n^o(t)]}{q_{max}^o} X_n^o(t) \quad (53)$$

The queues of private data have similar inequalities above. Furthermore, we can derive that:

$$\begin{aligned} \Delta L(\mathbf{Q}) - V \sum_{n=1}^N \mathbb{E}\{\theta_n \mu_n^p + \varphi_n \mu_n^o | \mathbf{Q}\} &\leq B - Q_0 \mathbb{E}\{R_0^{PU} - D_0^{PU} | \mathbf{Q}\} - Y \mathbb{E}\{P_{avg} - E | \mathbf{Q}\} + \sum_{n=1}^N \left\{ \frac{X_n^o(C_{max}^{o2} + \mu_{max}^2)}{2q_{max}^o} \right. \\ &+ \frac{X_n^p(C_{max}^{p2} + D_{max}^2)}{2q_{max}^p} - \frac{X_n^p Q_n^p}{q_{max}^p} \mathbb{E}\{R_n^p - T_n^p | \mathbf{Q}\} - \\ &\frac{X_n^o Q_n^o}{q_{max}^o} \mathbb{E}\{R_n^o - T_n^o | \mathbf{Q}\} - (1 - \frac{D_{max}}{q_{max}^p}) X_n^p \mathbb{E}\{T_n^p - \mu_n^p | \mathbf{Q}\} - \\ &(1 - \frac{\mu_{max}}{q_{max}^o}) X_n^o \mathbb{E}\{T_n^o - \mu_n^o | \mathbf{Q}\} - Z_n \mathbb{E}\{\rho_n \mu_n^o - Q_n^o | \mathbf{Q}\} \\ &\left. - V \mathbb{E}\{\theta_n \mu_n^p + \varphi_n \mu_n^o | \mathbf{Q}\} \right\} \quad (54) \end{aligned}$$

where $B = \frac{1}{2}(D_{max}^{PU2} + R_{0max}^2 + P_{max}^2 + P_{avg}^2) + N[\frac{1}{2}q_{max}^o \mu_{max} + (1 - \frac{\mu_{max}}{q_{max}^o})\mu_{max}^2 + (1 - \frac{D_{max}}{q_{max}^p})D_{max}^2 + \frac{1}{2}q_{max}^p D_{max}] + \frac{1}{2} \sum_{n=1}^N (\rho_n^2 \mu_{max}^2 + q_{max}^{o2})$ and $C_{max}^p = \max_n \{R_n^p\}$, $C_{max}^o = \max_n \{R_n^o\}$, $R_{0max} = \max \{R_0^{PU}\}$. Here we can find that our algorithm minimizes the right hand side (RHS) of (54).

In order to prove *Theorem 2*, we introduce *Lemma 3*.

Lemma 3: For any feasible rate vector $\mathbf{t} \in \Upsilon$, there exists a \mathbf{a} -only policy SR which stabilizes the network with the data admitted rate vector, $(\mu_{n,SR}^p(t), t_{n,SR}^p(t), \mu_{n,SR}^o(t), t_{n,SR}^o(t))$, and the service vector, $(R_{n,SR}^p(t), R_{n,SR}^o(t))$, independent of data queues. For all t and all $n \in \{1, 2, \dots, N\}$, the flow constraints are satisfied:

$$\begin{aligned} \mathbb{E}\{\mu_{n,SR}^o(t)\} &= \mathbb{E}\{T_{n,SR}^o(t)\} = \mathbb{E}\{R_{n,SR}^o\} \\ \mathbb{E}\{\mu_{n,SR}^p(t)\} &= \mathbb{E}\{T_{n,SR}^p(t)\} = \mathbb{E}\{R_{n,SR}^p\} \end{aligned}$$

Notice that, the stationary randomized policy SR makes decisions only depending on channel condition and independent of queue backlogs. Furthermore it may not fulfill the delay constraints. Similar proof of \mathbf{a} -only policy is given in [17] and the proof of *Lemma 3* is omitted here.

We can control the admitted rate of \mathbf{t} ranging from $\mathbf{t}^*(\epsilon)$ to $\mathbf{t}^*(\epsilon) + \epsilon$ arbitrarily and resulting in that both $\mathbf{t}^*(\epsilon)$ and $\mathbf{t}^*(\epsilon) + \epsilon$ are within Υ . It is assumed that the sufficient condition of full overlay optimum (21) is satisfied in our system, so according to *Lemma 2*, full overlay can achieve the optimal result. Besides, according to *Lemma 3*, it is true that there exist two different \mathbf{a} -only policies SR_1 and SR_2 which satisfy:

$$\mathbb{E}\{T_{n,SR_1}^o\} = \mathbb{E}\{R_{n,SR_1}^o\} = \mathbb{E}\{\mu_{n,SR_1}^o\} = t_{n,*}^{o,*}(\epsilon) \quad (55)$$

$$\mathbb{E}\{T_{n,SR_1}^p\} = \mathbb{E}\{R_{n,SR_1}^p\} = \mathbb{E}\{\mu_{n,SR_1}^p\} = t_{n,*}^{p,*}(\epsilon) \quad (56)$$

$$\mathbb{E}\{T_{n,SR_2}^o\} = \mathbb{E}\{R_{n,SR_2}^o\} = \mathbb{E}\{\mu_{n,SR_2}^o\} = t_{n,*}^{o,*}(\epsilon) + \epsilon \quad (57)$$

$$\mathbb{E}\{T_{n,SR_2}^p\} = \mathbb{E}\{R_{n,SR_2}^p\} = \mathbb{E}\{\mu_{n,SR_2}^p\} = t_{n,*}^{p,*}(\epsilon) + \epsilon \quad (58)$$

In addition, for policy SR_1 and SR_2 , it is easy to prove that:

$$\mathbb{E}\{R_{0,SR_1}^{PU}\} \geq \lambda_0 + \epsilon \quad (59)$$

$$\mathbb{E}\{E_{SR_2}\} \leq P_{avg} - \epsilon \quad (60)$$

Our algorithm minimizes RHS of (54) among all possible policies including SR policy, thus we can get :

$$\begin{aligned} \Delta L(\mathbf{Q}) - V \sum_{n=1}^N \mathbb{E}\{\theta_n \mu_n^p + \varphi_n \mu_n^o\} &\leq B + Y \{\mathbb{E}\{E_{SR_2}\} - P_{avg}\} - Q_0 \{\mathbb{E}\{R_{0,SR_1}^{PU}\} - \lambda_0\} + \\ &\sum_{n=1}^N \left\{ Z_n Q_n^o + \frac{C_{max}^{o2} + \mu_{max}^2}{2q_{max}^o} X_n^o + \frac{C_{max}^{p2} + D_{max}^2}{2q_{max}^p} X_n^p + \right. \\ &\left[\frac{q_{max}^o - \mu_{max}}{q_{max}^o} X_n^o - Z_n \rho_n - V \varphi_n \right] X_n^o \mathbb{E}\{\mu_{n,SR_1}^o\} + \\ &\mathbb{E}\{T_{n,SR_2}^o\} \frac{X_n^o}{q_{max}^o} [Q_n^o + \mu_{max} - q_{max}^o] - \frac{X_n^o Q_n^o}{q_{max}^o} \mathbb{E}\{R_{n,SR_2}^o\} + \\ &\mathbb{E}\{T_{n,SR_2}^p\} \frac{X_n^p}{q_{max}^p} [Q_n^p - q_{max}^p + D_{max}] - \frac{X_n^p Q_n^p}{q_{max}^p} \mathbb{E}\{R_{n,SR_2}^p\} + \\ &\left. \mathbb{E}\{\mu_{n,SR_1}^p\} \left[\frac{q_{max}^p - D_{max}}{q_{max}^p} X_n^p - V \theta_n \right] \right\} \quad (61) \end{aligned}$$

After substituting (55)-(58), (59) and (60) into the RHS of (61) and transforming it, we can derive that:

$$\begin{aligned} \Delta L(Q) - V \sum_{n=1}^N \mathbb{E}\{\theta_n \mu_n^p + \varphi_n \mu_n^o\} &\leq B - \epsilon(Y + Q_0) - \\ &\sum \{t_{n,*}^{o,*}(\epsilon) \rho - q_{max}\} Z - V \sum_{n=1}^N \{ \varphi_n t_{n,*}^{o,*}(\epsilon) + \theta_n t_{n,*}^{p,*}(\epsilon) \} - \\ &\sum_{n=1}^N \frac{X_n^o}{q_{max}^o} \{ \epsilon(q_{max}^o - \mu_{max}) - \frac{C_{max}^{o2} + \mu_{max}^2}{2} \} - \\ &\sum_{n=1}^N \frac{X_n^p}{q_{max}^p} \{ \epsilon(q_{max}^p - D_{max}) - \frac{C_{max}^{p2} + D_{max}^2}{2} \} \quad (62) \end{aligned}$$

So when (44)-(46) hold, we can find $\epsilon_1 > 0$ that $\epsilon_1 \leq \frac{\epsilon(q_{max}^o - \mu_{max}) - \frac{C_{max}^{o2} + \mu_{max}^2}{2}}{q_{max}^o}$, $\epsilon_1 \leq t_{n,*}^{o,*}(\epsilon) \rho_n - q_{max}$ and $\epsilon_1 \leq \frac{\epsilon(q_{max}^p - D_{max}) - \frac{C_{max}^{p2} + D_{max}^2}{2}}{q_{max}^p}$. Thus:

$$\begin{aligned} \Delta L(Q) - V \sum_{n=1}^N \mathbb{E}\{\theta_n \mu_n^p + \varphi_n \mu_n^o\} &\leq B - V \sum_{n=1}^N \{ \varphi_n t_{n,*}^{o,*}(\epsilon) + \\ &\theta_n t_{n,*}^{p,*}(\epsilon) \} - \sigma \left(\sum_{n=1}^N \{ X_n^o + X_n^p + Z_n \} + Y + Q_0 \right) \quad (63) \end{aligned}$$

where $\sigma = \min\{\epsilon, \epsilon_1\}$.

It can be got that when (44), (45) and (46) hold, (48) and

$$\liminf_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \sum_{n=1}^N \{\theta_n \mu_n^p(\tau) + \varphi_n \mu_n^o(\tau)\} \geq \sum_{n=1}^N \{\varphi_n t_n^{o,*}(\epsilon) + \theta_n t_n^{p,*}(\epsilon)\} - \frac{B}{V} \quad (64)$$

are satisfied by applying the theorem of Lyapunov Optimization, *Theorem 4.2* in [47], on (63) directly. Furthermore, (48) implies that (10) and (11) hold since X_n^o and X_n^p are kept stable. So after substituting (10) and (11) into (64), (47) holds. Hence the proof of *Theorem 2* is completed.

APPENDIX C PROOF OF LEMMA 1

From the constraint $r_0^{PU} = \lambda_0$, we can represent κ as a function of p_n^m and substitute it into (16). Then, the optimum solution can be found by solving:

$$\begin{aligned} \max_{p_n^m} \quad & \lambda_0 f(\mathbf{P}^{SU}) + \sum_{n=1}^N \sum_{m=1}^M \zeta_n \{(\theta_n - \varphi_n) [\log_2(1 + a_n^m p_n^m) \\ & - \log_2(1 + b_n^m p_n^m)]^+ + \varphi_n W \log_2(1 + a_n^m p_n^m)\} \\ \text{s.t.} \quad & 0 \leq \sum_{\forall n, \forall m} p_n^m \leq P_{max} \end{aligned} \quad (65)$$

where $f(\mathbf{P}^{SU}) = \sum_{n=1}^N \{(\theta_n - \varphi_n) \zeta_n \{ \sum_{m \in \Gamma_0^{PU}} [\log_2(1 + \frac{p_n^m a_n^m}{1 + P_0^m a_n^m}) - \log_2(1 + \frac{p_n^m b_n^m}{1 + P_0^m b_n^m})]^+ - \sum_{m \in \Gamma_0^{PU}} [\log_2(1 + \frac{a_n^m p_n^m}{1 + P_0^m a_n^m}) - \log_2(1 + \frac{b_n^m p_n^m}{1 + P_0^m b_n^m})]^+ \} + \varphi_n \sum_{m \in \Gamma_0^{PU}} [\log_2(1 + \frac{a_n^m p_n^m}{1 + P_0^m a_n^m}) - \log_2(1 + \frac{b_n^m p_n^m}{1 + P_0^m b_n^m})]^+ \} / \sum_{m \in \Gamma_0^{PU}} \{ \log_2(1 + \frac{A_0^m P_0^m}{1 + a_0^m p_n^m}) \}$ and we denote the denominator of $f(p)$ as Δ_d .

It is reasonable to make an approximation of $f(\mathbf{P}^{SU})$ under the assumption that PU is in a high SINR region such that $\log_2(1 + a_0^m p_n^m + A_0^m P_0^m) \approx \log_2(1 + A_0^m P_0^m)$.

One sufficient condition of full overlay scheme achieving the optimal solution of (65) is that $\kappa = 1$ makes the objective of (66) greater than any other $\kappa \geq 0$.

$$\begin{aligned} \max_{0 < p_n^m \leq P^*} \quad & \lambda_0 f(\mathbf{P}^{SU}) + \sum_{n=1}^N \sum_{m=1}^M \zeta_n \{(\theta_n - \varphi_n) [\log_2(1 + a_n^m p_n^m) \\ & - \log_2(1 + b_n^m p_n^m)]^+ + \varphi_n W \log_2(1 + a_n^m p_n^m)\} \end{aligned} \quad (66)$$

where $P^* \leq P_{max}$ is any positive upper bound of p_n^m .

Thus we need to find a condition where the maximization solution of (66) is an increasing function of $p_n^m, \forall m \in \Gamma_0^{PU}$. As p_n^m increases, it will force κ to increase as well, eventually reaching $\kappa = 1$, which is full overlay.

Firstly, we analyze the situation when $\zeta_n = 1$ and $a_n^m > b_n^m$. In this situation, security transmission happens with positive private transmission rate and $[\log(1 + p_n^m a_n^m) - \log(1 + p_n^m b_n^m)]^+ = \log(1 + p_n^m a_n^m) - \log(1 + p_n^m b_n^m)$. We derive the first derivative of the objective of (66):

$$(\theta_n - \varphi_n) \left(\frac{a_n^m}{1 + a_n^m p_n^m} - \frac{b_n^m}{1 + b_n^m p_n^m} \right) +$$

$$\varphi_n W \frac{a_n^m}{1 + a_n^m p_n^m} + \lambda_0 \frac{\partial f(\mathbf{P}^{SU})}{\partial p_n^m} \quad (67)$$

Then for any $m \in \Gamma_0^{PU}$, we derive that:

$$\begin{aligned} \frac{\partial f(\mathbf{P}^{SU})}{\partial p_n^m} = & \frac{a_{0S}^m}{\Delta_d^2 (1 + a_{0S}^m p_n^m)} \{ \theta_n [\log_2 \frac{(1 + b_n^m p_n^m)}{(1 + a_n^m p_n^m)} + \\ & [\log_2(1 + \frac{p_n^m a_n^m}{1 + P_0^m a_n^m}) - \log_2(1 + \frac{p_n^m b_n^m}{1 + P_0^m b_n^m})]] + \\ & \varphi_n [\log_2(1 + \frac{p_n^m a_n^m}{1 + P_0^m a_n^m}) - \log_2(1 + \frac{p_n^m b_n^m}{1 + P_0^m b_n^m})]] + \\ & \frac{1}{\Delta_d} \{ \varphi_n [\frac{b_n^m}{1 + P_0^m b_n^m + b_n^m p_n^m} - \frac{b_n^m}{1 + b_n^m p_n^m}] + \\ & \theta_n [(\frac{b_n^m}{1 + b_n^m p_n^m} - \frac{a_n^m}{1 + a_n^m p_n^m}) + (\frac{a_n^m}{1 + P_0^m a_n^m + a_n^m p_n^m} - \\ & \frac{b_n^m}{1 + P_0^m b_n^m + b_n^m p_n^m})] \} \} \end{aligned} \quad (68)$$

Substituting (68) into (67), we can get the numeration of the first derivative of the objective of (66):

$$\begin{aligned} & \Delta_d [\theta_n (\Delta_d - \lambda_0) \left(\frac{a_n^m}{1 + a_n^m p_n^m} - \frac{b_n^m}{1 + b_n^m p_n^m} \right) + \\ & \theta_n \lambda_0 \left(\frac{a_n^m}{1 + P_0^m a_n^m + a_n^m p_n^m} - \frac{b_n^m}{1 + P_0^m b_n^m + b_n^m p_n^m} \right) + \\ & \varphi_n (\Delta_d - \lambda_0) \frac{b_n^m}{1 + b_n^m p_n^m} + \varphi_n \lambda_0 \frac{b_n^m}{1 + P_0^m b_n^m + b_n^m p_n^m}] + \\ & \varphi_n [\log_2(1 + \frac{b_n^m p_n^m}{1 + P_0^m b_n^m}) - \log_2(1 + b_n^m p_n^m)] + \\ & \frac{a_{0S}^m}{1 + a_{0S}^m p_n^m} \lambda_0 \{ \theta_n [\log_2(1 + \frac{p_n^m a_n^m}{1 + P_0^m a_n^m}) - \\ & \log_2(1 + \frac{p_n^m b_n^m}{1 + P_0^m b_n^m})] \} - \log_2 \frac{(1 + a_n^m p_n^m)}{(1 + b_n^m p_n^m)} \\ & \geq \Delta_d [\theta_n \lambda_0 \left(\frac{a_n^m}{1 + P_0^m a_n^m + a_n^m p_n^m} - \frac{b_n^m}{1 + P_0^m b_n^m + b_n^m p_n^m} \right) + \\ & \varphi_n \lambda_0 \frac{b_n^m}{1 + P_0^m b_n^m + b_n^m p_n^m}] - a_{0S}^m \lambda_0 \{ \theta_n [\log_2(1 + a_n^m p_n^m) - \\ & \log_2(1 + b_n^m p_n^m)] + \varphi_n \log_2(1 + b_n^m p_n^m) \} \\ & \geq \theta_n \lambda_0 \left(\frac{a_n^m}{1 + P_0^m a_n^m + a_n^m p_n^m} - \frac{b_n^m}{1 + P_0^m b_n^m + b_n^m p_n^m} \right) + \\ & \varphi_n \lambda_0 \frac{b_n^m}{1 + P_0^m b_n^m + b_n^m p_n^m} - a_{0S}^m \lambda_0 \varphi_n \log_2(1 + b_n^m p_n^m) - \\ & a_{0S}^m \lambda_0 \theta_n [\log_2(1 + a_n^m p_n^m) - \log_2(1 + b_n^m p_n^m)] \end{aligned} \quad (69)$$

The last inequality in (69) holds under the assumption that $A_0^m \gg a_{0S}^m$.

For any pair of (θ_n, φ_n) , the RHS of the last inequality of (69) is greater than zero if and only if

$$\left(\frac{a_n^m}{1 + P_0^m a_n^m + a_n^m p_n^m} - \frac{b_n^m}{1 + P_0^m b_n^m + b_n^m p_n^m} \right) - a_{0S}^m [\log_2(1 + a_n^m p_n^m) - \log_2(1 + b_n^m p_n^m)] \geq 0 \quad (70)$$

$$\frac{b_n^m}{1 + P_0^m b_n^m + b_n^m p_n^m} - a_{0S}^m \log_2(1 + b_n^m p_n^m) \geq 0 \quad (71)$$

Sufficient conditions for full overlay are:

$$a_{0S}^m \leq \bar{C}_{mn}^3 = \frac{\frac{a_n^m}{1+P_0^m a_{nP}^m + a_n^m p_n^m} - \frac{b_n^m}{1+P_0^m b_{nP}^m + b_n^m p_n^m}}{\log_2(1 + a_n^m p_n^m) - \log_2(1 + b_n^m p_n^m)} \quad (72)$$

$$a_{0S}^m \leq \bar{C}_{mn}^1 = \frac{\frac{b_n^m}{1+P_0^m a_{nP}^m + a_n^m p_n^m}}{\log_2(1 + b_n^m p_n^m)} \quad (73)$$

So the sufficient condition of full overlay when $\zeta_n = 1$ and $a_n^m > b_n^m$ is:

$$a_{0S}^m \leq \min\{\bar{C}_{nm}^3, \bar{C}_{nm}^1\} \quad (74)$$

And for $\zeta_n = 0$ or $a_n^m \leq b_n^m$ we can get similar conclusion and omit the process here. The sufficient condition is:

$$a_{0S}^m \leq \bar{C}_{nm}^2 = \frac{\frac{a_n^m}{1+P_0^m a_{nP}^m + a_n^m p_n^m}}{\log_2(1 + a_n^m p_n^m)} \quad (75)$$

It is easy to find that for any $a_{0S}^m \leq \min\{\bar{C}_{nm}^1, \bar{C}_{nm}^2\}$, a_{0S}^m is definitely no greater than \bar{C}_{nm}^3 . So for any m , the sufficient condition for the optimum of user n accessing this subcarrier m in full overlay mode is $a_{0S}^m \leq \min\{\bar{C}_{nm}^1, \bar{C}_{nm}^2\}$.

For $P_{max} > p_n^m$, $C_{nm}^1 \leq \bar{C}_{nm}^1$ and $C_{nm}^2 \leq \bar{C}_{nm}^2$ hold. Hence the results in Lemma 1 are proved.

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